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Total and Form Drag Friction Factors for the Turbulent Flow of Air Through Packed and Distended Beds of Spheres

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Several attempts have been made to determine the form and shear drag contributions to the overall friction factors for the flow of gases past spherical surfaces. Fage (*4*) has measured the static pressure around a single sphere 6 in. in diam. mounted in an open jet tunnel. His results indicate that the form drag caused by the static pressure on the surface of the sphere is the major component of the total drag, and that the contribution due to shear drag resulting from skin friction is relatively small. Similarly, Yen and Thodos (*8*) have determined, with a sensitive micromanometer, static-pressure profiles for the flow of air past a single celite sphere, 2.02 in. in diam. In his study, the total drag was also determined by measuring with an analytical balance the force exerted by air flowing past the sphere. Again, the results of this study indicate that the friction factors for total drag and for form drag resulting from the turbulent flow of air are of approximately the same order of magnitude. Flachsbart (*3*) has measured the pressure distribution around a single sphere for turbulent and nonturbulent conditions.

Although many investigators have measured the total pressure drop resulting from the flow of gases through packed beds of spherical particles (*1*, *2*, *5*), no attempts were made to resolve the resulting friction factors into their form-drag and shear-drag components. Therefore, in this study an effort has been made to account for the form and shear-drag contributions to the total drag for fixed beds by measuring the static pressure around the surfaces of test spheres contained in packed and distended beds of smooth plastic spheres.

EXPERIMENTAL PROCEDURE

The experimental apparatus included a vertical wind tunnel, 14 in. in diam. and 26 ft. high, which was connected to the suction end of a centrifugal blower. A sliding window near the blower permitted the flow of air to be varied from 2.5 to 13.4 ft./sec. The test section containing the bed of spheres was 22 in. high and was placed 10 ft. below the inlet of the wind tunnel. The air velocity was measured 26 in. above the test section with a sliding pitot tube which was connected to a micromanometer capable of detecting pressure differences of 0.0001 in. of manometer fluid (isobutyl alcohol).

The packed beds contained spheres, 1.23 in. in diam., which were held together in a specific geometric configuration with fine rigid wires, 0.018 in. in diam. The distended beds were formed by arranging the spheres in a specific orientation and then separating them and holding them fixed in space with short lengths of the rigid wire. The wires were permanently fixed with epoxy resin in holes drilled into the spheres at the proper angles.

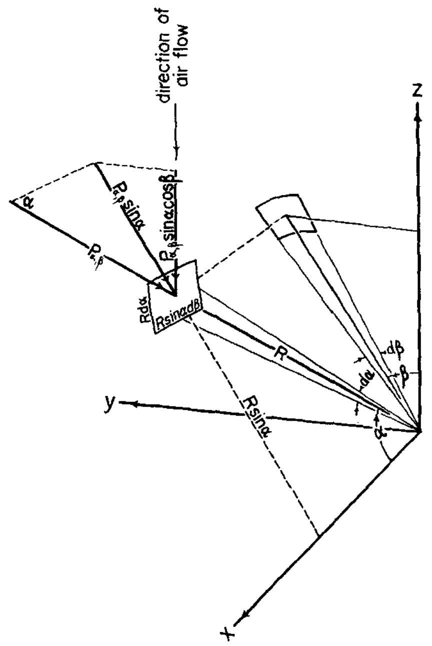

Fig. 1. Representation of the form drag force on an element of spherical surface rotated on an axis perpendicular to the direction of flow.

The packed and distended beds contained five layers of spheres and had the following geometric configurations and void fractions:

Orientation	Packed bed (ϵ)	Distended bed (ϵ)
Cubic	0.480	0.882
Body-centered cubic	0.345	0.615, 0.728
Face-centered cubic		0.743

Three test spheres were utilized for the static-pressure measurements. Each test sphere could be placed in the center of the middle layer of each packed and distended bed. A hollow stainless steel tube was inserted into a hole drilled to the center of each test sphere and was used to support the sphere in the bed. Additional holes were drilled into the three test spheres at offset angles to the first hole of $\alpha = 90$ deg. for the first sphere, $\alpha = 60$ deg. for the second sphere, and $\alpha = 30$ deg. for the third sphere. These holes served as the pitot end for the measurement of the static pressure on the surface of the spheres. The hollow tube was held fixed in the center of a rotatable dial mounted on a solid support. The tube was then attached to one end of the micro-manometer, whose other end was connected to a fixed pressure tap upstream from the bed. By turning the dial and thus rotating the sphere about a horizontal axis, static-pressure measurements were obtained at 10 deg. intervals as the pitot opening traversed a great circle for the sphere having an offset angle, $\alpha = 90$ deg., and minor circles for the two other spheres. The measurements were made for three test spheres to account for the effects of the irregular flow pattern of the air in the interstices of the spheres of the bed.

DEVELOPMENT OF FORM DRAG RELATIONSHIP FOR TEST SPHERE

In Figure 1 a schematic diagram representing a surface element of a test sphere is presented. For an offset angle, α from the horizontal axis of rotation, x, and an angle of rotation, β, from the xz-plane, the area of an element on the surface of a test sphere of radius R is

$$dA = (R \sin \alpha \, d\beta)(R d\alpha) \quad (1)$$

The form drag force exerted in the direction of flow on the element dA is

$$dF_p = (P_{\alpha,\beta} - P^*) \sin \alpha \cos \beta dA \quad (2)$$

where $P_{\alpha,\beta}$ is the normal pressure and P^* is the static pressure of the undisturbed air at a reference point upstream from the bed. When Equations (1) and (2) are combined

$$dF_p = (P_{\alpha,\beta} - P^*) R^2 \sin^2 \alpha \cos \beta \, d\beta d\alpha \quad (3)$$

Therefore, Equation (3) can be integrated to produce the following expression for the form drag force on the sphere:

$$F_p = R^2 \int_0^{2\pi} \int_0^{\pi} (P_{\alpha,\beta} - P^*) \sin^2 \alpha \cos \beta \, d\beta d\alpha \quad (4)$$

In this investigation, the offset angle, α, was varied only from 0 to $\frac{\pi}{2}$, and a symmetrical pressure distribution over the other half of the sphere was assumed.

The form drag force on the surface of a test sphere can also be related to the surface area of the sphere, $A_s = 4\pi R^2$, and the kinetic energy of the flowing air, as follows:

$$F_p = f_p \, (4\pi R^2) \frac{\rho u^2}{2g_c} \quad (5)$$

where f_p is the contribution to the total friction factor owing to form drag. By combining Equations (4) and (5), the form drag friction factor for this case becomes

$$f_p = \frac{1}{4\pi} \int_{\alpha=0}^{\alpha=\pi} \int_{\beta=0}^{\beta=2\pi} \sin^2 \alpha \frac{P_{\alpha,\beta} - P^*}{\rho u^2/2g_c} \cos \beta \, d\beta d\alpha \quad (6)$$

Therefore, form drag friction factors for the packed and distended beds were established by determining for each test sphere (having a constant value of α) the static-pressure difference as the angle of rotation, β, varied from 0 to 2π, and performing the necessary integrations graphically.

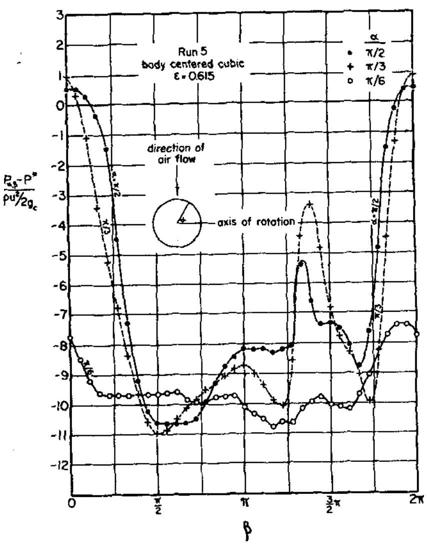

Fig. 2. Normal pressure distribution measurements for the flow of air past the surface of a sphere located in the center of a distended bed.

FORM DRAG MEASUREMENTS

Fifty-one experimental runs were conducted on the packed and distended beds of spheres to obtain information on the variation of the static-pressure difference, $P_{\alpha,\beta} - P^*$, with angle of rotation for each test sphere. The experimental data can be found elsewhere (7). The results of a typical run, in which a body-centered cubic distended bed having a void fraction of $\epsilon = 0.615$ was utilized, are presented in Table 1 for only angles of rotation of 0 deg. $\leq \beta \leq 90$ deg. In Figure 2, the resulting values of the ratio of the static-pressure difference divided by the kinetic energy are plotted vs. the corresponding angle of rotation for each offset angle. The irregularity in this figure for angles of

TABLE 1. RESULTS OF FORM DRAG MEASUREMENTS FOR A TYPICAL RUN

(Run 5: Body-centered cubic, distended bed, $\epsilon = 0.615$)

$$\frac{\rho u^2}{2g_c} = 0.0518 \text{ lb.}_f\text{/sq. ft} \qquad \frac{D_p G}{\mu} = 4{,}020$$

	$\alpha = \pi/2$		$\alpha = \pi/3$		$\alpha = \pi/6$	
β	$P_{\alpha,\beta}$-P^* lb.$_f$/sq. ft.	$\frac{P_{\alpha,\beta}-P^*}{\rho u^2/2g_c}$	$P_{\alpha,\beta}$-P^* lb.$_f$/sq. ft.	$\frac{P_{\alpha,\beta}-P^*}{\rho u^2/2g_c}$	$P_{\alpha,\beta}$-P^* lb.$_f$/sq. ft.	$\frac{P_{\alpha,\beta}-P^*}{\rho u^2/2g_c}$
0°	+0.0252	+0.486	+0.0463	+0.894	−0.4016	−7.75
10	+0.0261	+0.504	+0.0157	+0.303	−0.4388	−8.47
20	+0.0132	+0.255	−0.0579	−1.118	−0.4773	−9.21
30	−0.0190	−0.367	−0.1778	−3.43	−0.4971	−9.60
40	−0.0753	−1.45	−0.2713	−5.24	−0.5029	−9.71
50	−0.2345	−4.53	−0.3511	−6.78	−0.5034	−9.72
60	−0.3784	−7.31	−0.4339	−8.38	−0.5025	−9.70
70	−0.4773	−9.21	−0.4996	−9.64	−0.5017	−9.69
80	−0.5311	−10.25	−0.5408	−10.44	−0.5009	−9.67
90	−0.5501	−10.62	−0.5670	−10.95	−0.5013	−9.68

rotation of approximately $\beta = \frac{3}{2}\pi$ can be attributed to the presence of irregular vortices in the flow pattern around the sphere. The information in this figure was utilized to produce points for the second graphical integration required by Equation (6) and performed in Figure 3 for this run. In Figure 3, the offset angle, α, varies from 0 to $\pi/2$, and the area under the resulting curve was doubled (assuming symmetry over both halves of the sphere) to produce the form drag friction factor, $f_p = 1.17$, for this run. The calculated values of f_p for all the runs of this investigation are presented elsewhere (7).

Friction factors due to form drag were plotted against the corresponding Reynolds number, $N_{Re} = D_p G/\mu$, on log-log coordinates to produce separate relationships for each void fraction. For a Reynolds number, $N_{Re} =$ 4,000, values of f_p obtained from this plot were plotted against the corresponding void fraction on log-log coordinates. A straight line having a slope of −10/3 resulted, suggesting that the product, $f_p \epsilon^{10/3}$, is a function only of Reynolds number and is independent of the geometric orientation of the spheres of the bed.

TOTAL FRICTION FACTORS

Wentz and Thodos (6) have measured the overall pressure drop through each packed and distended bed used in the present investigation for the same flow conditions. In addition, they measured for these flow conditions the pressure drop across the middle layer containing the test sphere for each distended bed. From these measurements, Wentz and Thodos developed a relationship for the ratio of the total friction factor for the entire bed, f'_t, to the total friction factor for the middle layer, f_t, which was found to be independent of the Reynolds number and void fraction of the bed. This relationship can be expressed as follows:

$$\frac{f'_t}{f_t} = 1.13 \qquad (7)$$

Therefore, for packed beds for which the pressure drop across a single layer cannot be easily measured, the total friction factor for the middle layer, f_t, can be calculated from the overall pressure drop through the bed from

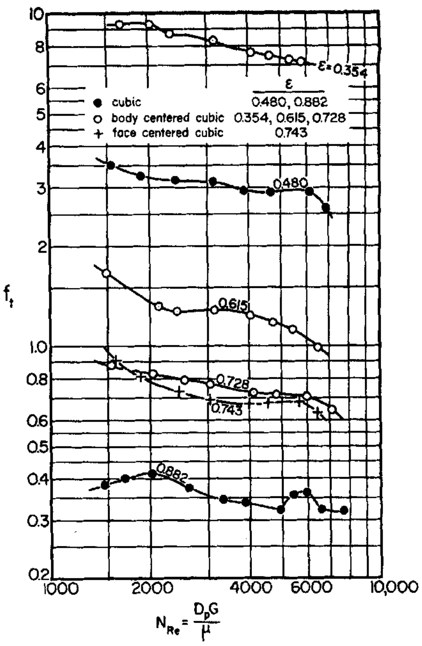

Fig. 4. Relationships between the friction factor for total drag, f_t, and Reynolds number for three geometrical orientations of spheres having different void fractions.

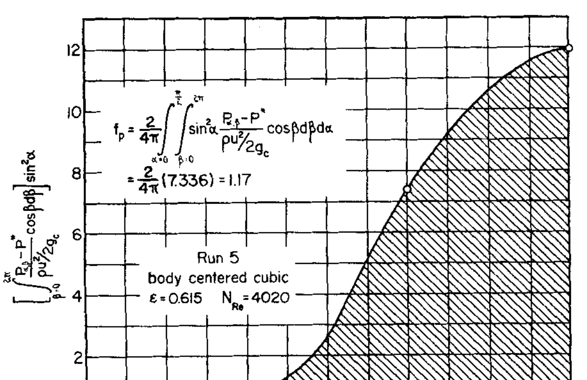

Fig. 3. Integration of normal pressure distribution data over the surface of a sphere to obtain f_p, the friction factor for form drag.

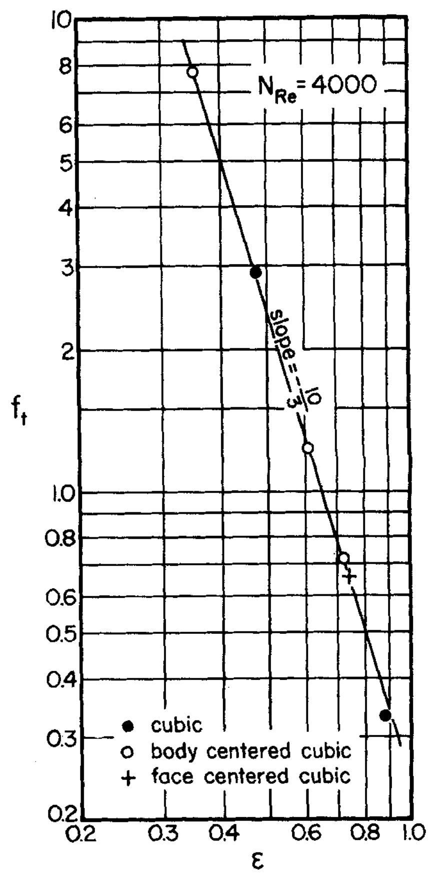

Fig. 5. Relationship between friction factor for total drag, f_t, and void fraction, ϵ (N_{Re} = 4,000).

Equation (7) and the following relationship:

$$A_c \, \Delta P = f'_t \, A_s \frac{\rho u^2}{2g_c} \qquad (8)$$

where ΔP is the pressure drop through the bed.

As a result, the previous experimental data enabled the determination of the friction factors, f_t, for each packed and distended bed investigated. The calculated value of f_t for each run is presented elsewhere (7). In Figure 4, the resulting total friction factors for the middle layer are plotted vs. the corresponding Reynolds number on log-log coordinates to again produce separate relationships for each void fraction. Following a procedure similar to that utilized for the form drag friction factors, values of f_t obtained from Figure 4 for a Reynolds number, $N_{Re} = 4{,}000$, were plotted against the corresponding void fraction on log-log coordinates as shown in Figure 5. Once again, a straight line having a slope of $-10/3$ resulted, indicating that the total friction factors are also independent of the geometric arrangement of the spheres in the bed.

From Equation (8), the following relationship for the total friction factor, f_t, can be developed for a fixed bed of spheres:

$$6f_t = \frac{\Delta P}{\rho u^2/2g_c} \frac{D_p}{L} \frac{1}{1-\epsilon} \qquad (9)$$

Figure 5 indicates that the product, $f_t \, \epsilon^{10/3}$, should be dependent only on Reynolds number. Therefore, Equation (9) can be multiplied by $\epsilon^{10/3}$ to produce

$$\frac{\Delta P}{\rho u^2/2g_c} \frac{D_p}{L} \frac{\epsilon^{10/3}}{1-\epsilon} = \varphi\left(\frac{D_p G}{\mu}\right) \qquad (10)$$

Thus Equation (10) is similar to

$$\frac{\Delta P}{\rho u^2/2g_c} \frac{D_p}{L} \frac{\epsilon^3}{1-\epsilon} = \chi\left(\frac{D_p G}{\mu\,(1-\epsilon)}\right) \qquad (11)$$

proposed by Ergun for packed beds (2).

In Figure 6 the products, $f_p \, \epsilon^{10/3}$ and $f_t \, \epsilon^{10/3}$, are plotted against the Reynolds number, $N_{Re} = D_p \, G/\mu$, for all the packed and distended beds investigated. For the form drag friction factor, the data can be represented by

$$f_p \, \epsilon^{10/3} = \frac{0.856}{N_{Re}^{1/6}} \qquad (12)$$

Similarly, the total friction factor can be represented by the

$$f_t \, \epsilon^{10/3} = \frac{0.972}{N_{Re}^{1/6}} \qquad (13)$$

Equations (12) and (13) apply for Reynolds numbers in the range $1{,}500 < N_{Re} < 8{,}000$.

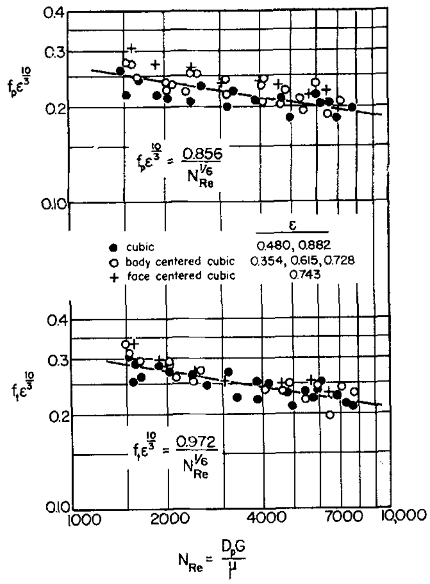

Fig. 6. Relationships between total and form drag friction factors and Reynolds number for packed and distended beds spheres having varying void fractions.

The work of Ergun (2) indicates that the total friction factor is only slightly dependent on Reynolds number in the range investigated in this study. Ergun utilized spheres of varying degrees of roughness, and therefore obtained friction factors considerably higher than those resulting from the present study in which spheres with very smooth surfaces were employed. Total friction factors resulting from the correlation of Carman (*1*) for the highest Reynolds numbers reported by him were found to be consistent with the corresponding values calculated from Equation (13). Total friction factors resulting from this equation were also found to compare very well with values obtained by Martin, McCabe, and Monrad (5) for the flow of fluids through packed beds of brass spheres, 5/16 and 5/8 in. in diam., arranged in various geometric configurations.

From the experimental data, friction factors due to shear drag, $f_s = f_t - f_p$, were obtained for each run. From these values, the product $f_s \, \epsilon^{10/3}$ was plotted against Reynolds number, and considerable scatter resulted. This scatter can be attributed to the fact that the total drag and form drag friction factors are of the same order of magnitude, and thus it is difficult to establish accurately their differences directly. However, since the friction factors resulting from Equations (12) and (13) represent average quantities, these equations can be used to estimate the shear drag contribution to the total drag to be $[(0.972 - 0.856)/0.972] \times 100 = 12\%$ for the turbulent flow of gases through packed and distended beds of spheres for Reynolds numbers in the range, $1{,}500 < N_{Re} < 8{,}000$.

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NOTATION

A = area
A_c = cross-sectional area of bed
A_s = surface area of spheres
D_p = sphere diameter, ft.
f_p = form drag friction factor
shear drag friction factor, $f_t - f_p$
f_t = total drag friction factor for test layer
f'_t = total drag friction factor for entire bed
g_c = conversion factor, 32.17 lb.$_m$ ft./lb.$_f$ sec.^{2}
F_p = form drag force
F_s = shear drag force
F_t = total drag force
G = superficial mass velocity, lb.$_m$/sec. sq. ft.
L = bed height, ft.
N_{Re} = Reynolds number, $D_p \, G/\mu$
P^* = static pressure of undisturbed air
$P_{\alpha,\beta}$ = normal pressure
R = radius of sphere
u = superficial gas velocity, ft./sec.

Greek Letters

α = offset angle between radius of surface element of sphere and the horizontal axis of rotation, x
β = angle of rotation from the xz-plane
ϵ = void fraction
μ = absolute viscosity, lb.$_m$/sec. ft.
π = 3.14159
ρ = density, lb.$_m$/cu. ft.

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